

COEFFICIENTS OF COUPLING FOR COUPLED-
CAVITY SLOW WAVE STRUCTURES

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THESIS

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FOR COUPLED-CAVITY SLOW WAVE STRUCTURES

by

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Thesis

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Coefficients of Coupling
for Coupled-Cavity Slow Wave Structures

by

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ABSTRACT

A method is presented for evaluating the coupling coefficients associated with loosely coupled cavity structures. The measure of coupling is expressed in terms of the shift in resonant frequency exhibited by a single aperture-coupled cavity. The frequency shift is determined from the resonant frequency of an equivalent circuit representing the aperture-coupled cavity. This method of analysis is applied to a specific waveguide cavity, coupled by a small circular aperture.

The results of this method are then presented for comparison with those obtained by two other techniques.

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I. INTRODUCTION

The "coupled-mode" theory has proved itself to be an important tool in analyzing the energy exchange phenomena between propagating waves and oscillating circuits. The application of this theory has simplified the analysis of many periodic slow-wave structures including the Traveling Wave Tube amplifier, parametric amplifier, and the Backward Wave Oscillator.

Louisell [9] gives a good description of how the coupled mode theory is applied in analyzing a device such as mentioned above. In essence, a coupled system is divided up into a number of isolated elements or unit cells. With the knowledge of the behavior of these isolated elements and of the interaction between them when coupled together, the behavior of the original structure can be approximated.

In a recent paper, J. B. Knorr [8] has applied this concept in analyzing a periodic waveguide constructed from loosely coupled microwave cavities. The periodic waveguide, treated as an infinite string of loosely coupled oscillators, is characterized by an infinite dimensional hermitian matrix. The resulting eigenvalues of this matrix possess several properties that are of particular importance in this present study. The eigenvalues, ω , are expressed as follows:

$$\omega = \omega_0 + 2h\cos\beta L \quad (1-1)$$

where β is the phase constant, L is the structure period, h is the coupling coefficient, and ω_0 is the natural frequency of oscillation of the individual cavities with the coupling aperture present but with the cavities uncoupled.

The ω - β diagram is shown by Eq. (1-1) to have branches which are sinusoidal. The character of the passband is completely determined by the matrix elements ω_0 and h .

To extend the range of usefulness of these eigenvalues into the quantitative domain, a procedure must be developed for calculating the values for h , the coupling coefficients. Referring to Eq. (1-1), it is evident that the second term, $2h\cos\beta L$, represents a shift in the cavity resonant frequency. The problem is therefore reduced to one of calculating this shift in resonant frequency.

In this thesis a basis is presented for calculating the coupling coefficients associated with a periodic waveguide structure. The original structure consists of identical cavities coupled by small circular apertures. An equivalent RLC circuit is derived for a single cavity representing a unit cell of this structure. The resonant frequency of this circuit is calculated and compared with the resonant frequency of the uncoupled cavity. The coupling coefficients are then determined by the difference in the two frequencies.

The organization of the material presented in this thesis is as follows: Section II discusses the periodic structure considered in this study and develops a single cavity model; Section III contains the derivation for the cavity equivalent circuit and the resulting expression for the resonant

frequency of the aperture coupled cavity; in Section IV the equivalent circuit method is applied to a specific cavity. Laboratory measurements as well as the results obtained by applying the method outlined in Appendix A are included for comparison; Section V presents the conclusions arrived at by this study and includes suggestions for further study; Appendix A contains an alternate method of solution which supplements the material in Sections II and IV.

The first part of the paper discusses the importance of the study of the history of the English language. It is argued that the study of the history of the English language is essential for a full understanding of the language and its development. The second part of the paper discusses the importance of the study of the history of the English language. It is argued that the study of the history of the English language is essential for a full understanding of the language and its development.

II. THEORETICAL MODEL DEVELOPMENT

The periodic waveguide shown in Fig. (2-1) illustrates the type of slow wave structure that will be considered in this study. The structure is assumed to consist of identical unit cells that resemble microwave cavities. The unit cells are coupled to each other by small circular apertures centered in the common walls.

The resonant cavities are first considered individually with the coupling apertures absent. The cavity resembles a shorted section of waveguide, as shown in Fig. (2-2).

The properties of resonant cavities arise fundamentally from the fact that, in order for the cavities to oscillate at all, the fields must satisfy the boundary conditions presented by the cavity walls. Solutions for the cavity fields are based upon the propagating modes of the waveguide in question. Although there are an infinite number of cavity resonances corresponding to each waveguide mode, it is assumed that for this discussion the cavities support a TE_{101} mode. The resonant frequency ω_{101} pertaining to the cavity shown in Fig. (2-2) is given by the following expression [1]:

$$\omega_{101} = c \left[\left(\frac{\pi}{a} \right)^2 + \left(\frac{\pi}{d} \right)^2 \right]^{1/2} \quad (2-1)$$

where a and d are cavity dimensions and c is the velocity of light.

The resonant frequency given by Eq. (2-1) is for a completely closed cavity. The presence of a coupling aperture

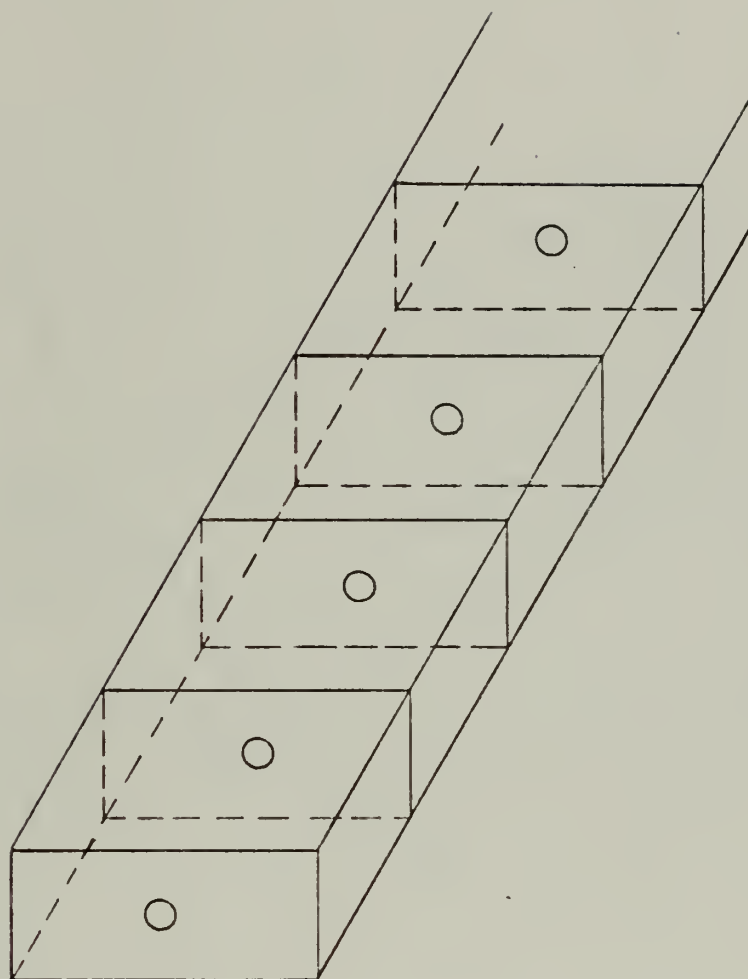


Figure (2-1). Coupled Cavity Periodic Structure

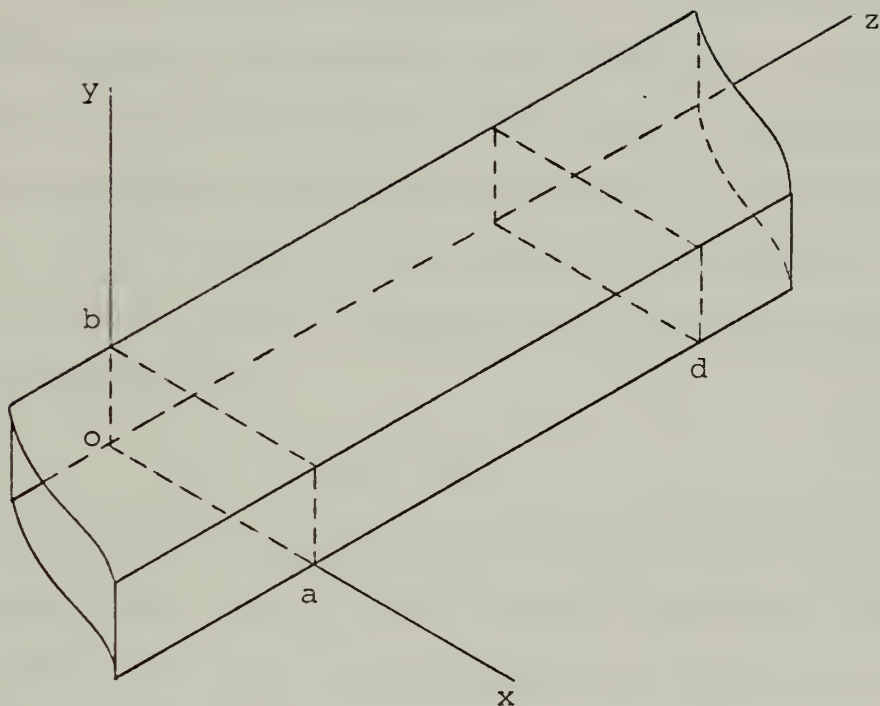


Figure (2-2). Unit Cell of a Periodic Structure

coupled, and are assumed to support a TE_{101} mode. The E- and H- field patterns indicated in Fig. (2-3a) and (2-3b) are for $\beta d = 0$ and $\beta d = \pi$, [3]. The resonant frequency for each of these field configurations is the same when there is no coupling and corresponds to the value given by Eq. (2-1).

The same two-cavity section is next considered with the coupling aperture present. Figure (2-4) illustrates the field configurations, again for $\beta d = 0$ and $\beta d = \pi$. For the cutoff point $\beta d = \pi$, shown in Fig. (2-4b), the presence of the coupling aperture does not perturb the field lines associated with either cavity. The resonant frequency for the coupled cavities remains unchanged. However, the field lines associated with the cutoff point $\beta d = 0$ have been perturbed, as shown in Fig. (2-4a). The magnetic lines in both cavities have been perturbed in the vicinity of the aperture, thus adding to the energy of the magnetic field. As a result, the resonant frequency decreases by an amount $\Delta\omega$, as given by Eqs. (2-2) and (2-3).

This passband behavior is illustrated in the ω - β diagram shown in Fig. (2-5). It is important to note that while this diagram is for the TE_{101} mode, there will be a passband for each mode of the unperturbed cavity. The ω'_0 noted on the diagram represents the resonant frequency of the coupled cavities. The shift in resonant frequencies, $\Delta\omega$, can be related to ω'_0 as follows:

$$\omega'_0 = \omega_0 + \Delta\omega \quad (2-4)$$

In this case $\Delta\omega$ is negative, which agrees with Eq. (2-2).

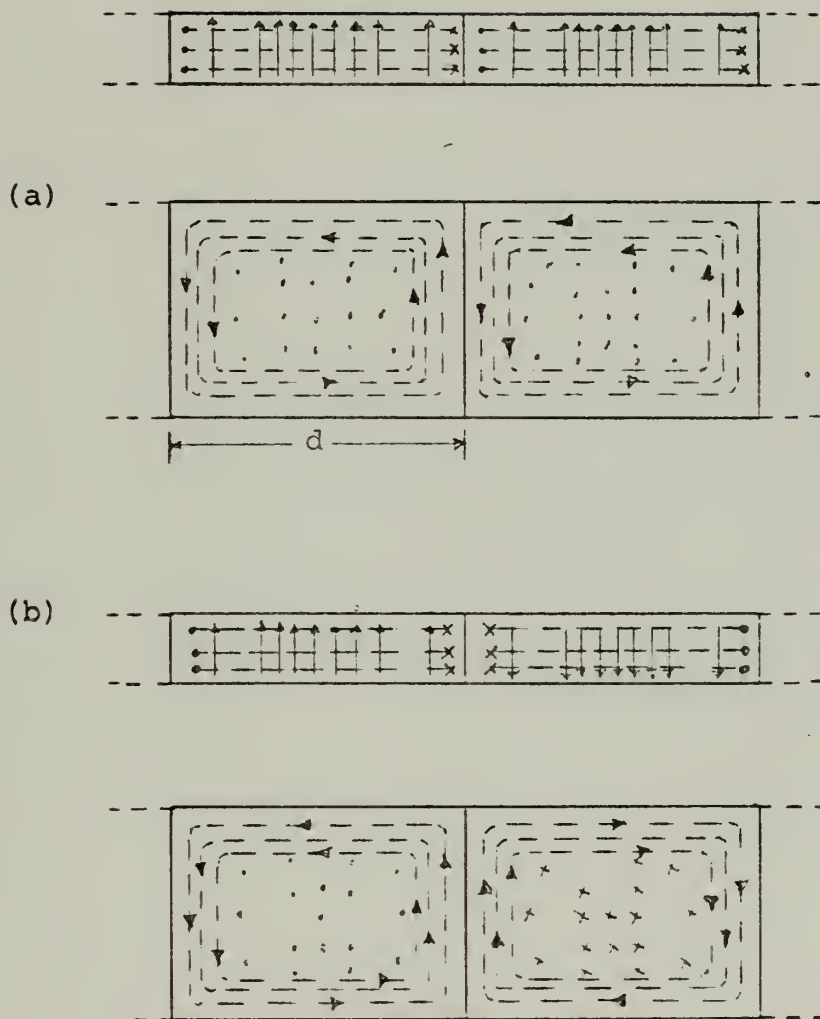


Figure (2-3). Two identical cavities with no coupling (solid lines = E-field; dashed lines = H-field). (a) Field pattern for $\beta d = 0$; (b) Field pattern for $\beta d = \pi$.

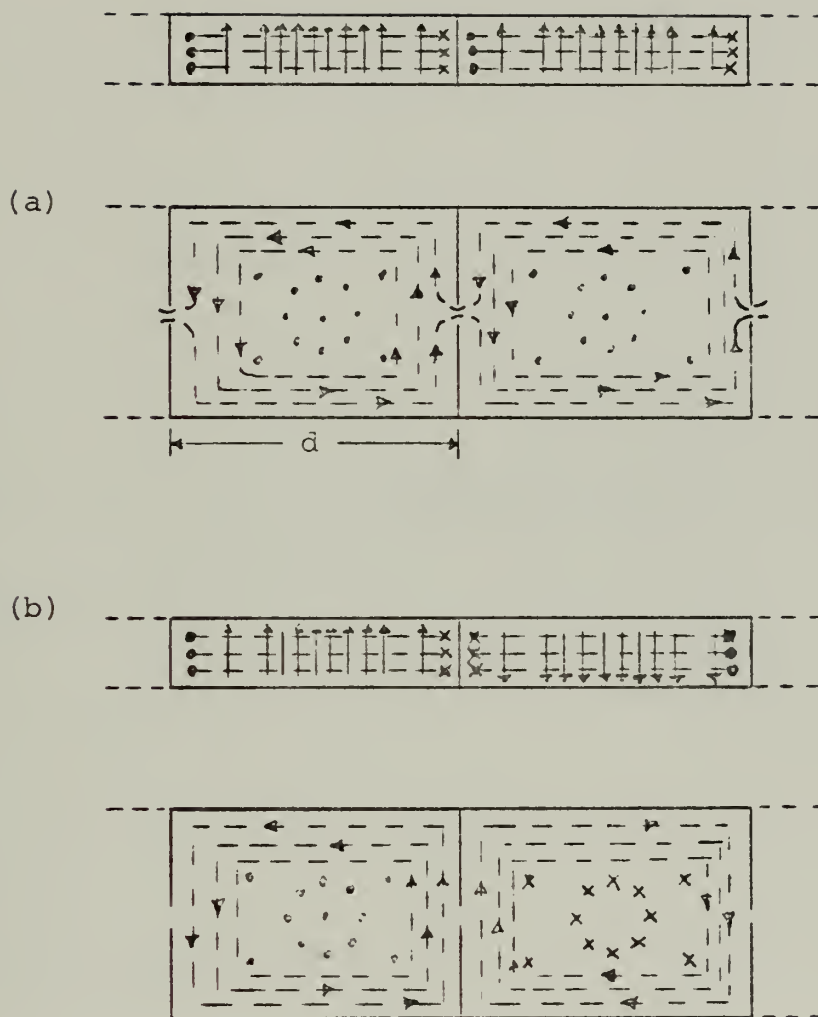


Figure (2-4). Cavities coupled by a circular aperture.
 (a) Field pattern for $\beta d = 0$;
 (b) Field pattern for $\beta d = \pi$.

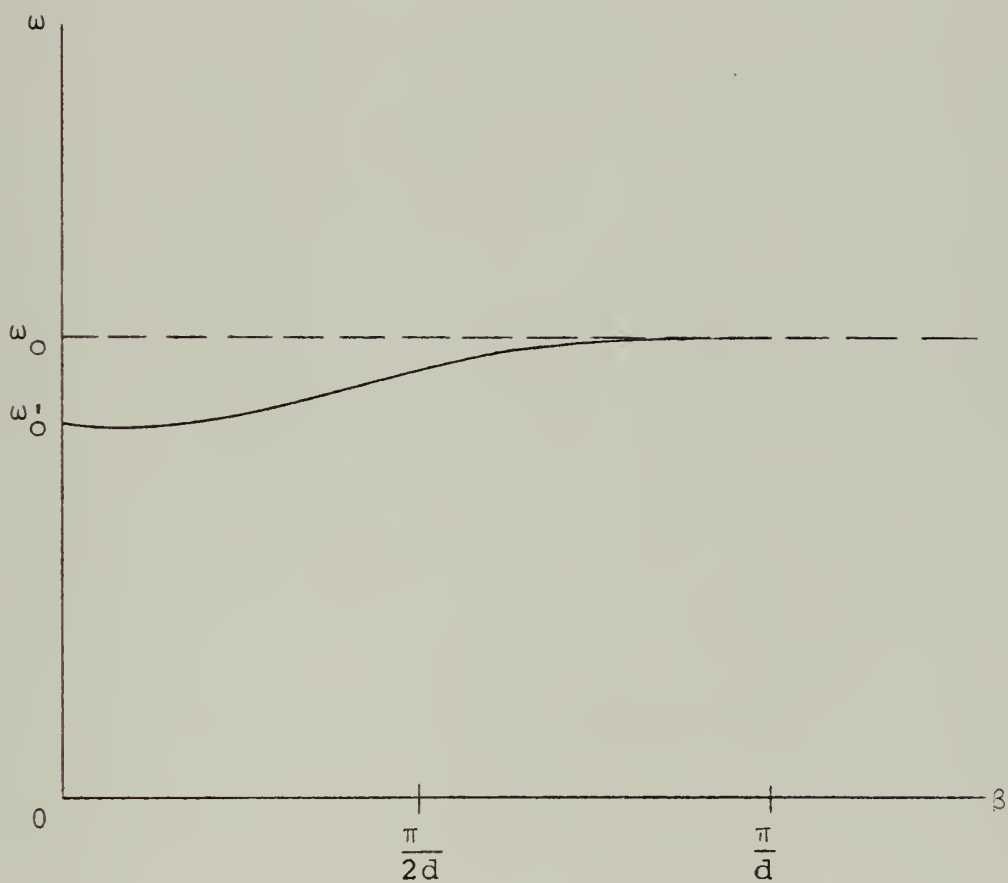


Figure (2-5). ω - β Diagram illustrating a cavity with inductive coupling.



This shift in frequency is related to the eigenvalues given by Eq. (1.1), as was mentioned previously. For the ω - β diagram shown in Fig. (2.5), $\Delta\omega$ is equal to $4h$, where h is the coupling coefficient.

$\Delta\omega$ will now be determined quantitatively by analyzing the single cavity model shown in Fig. (2-6).

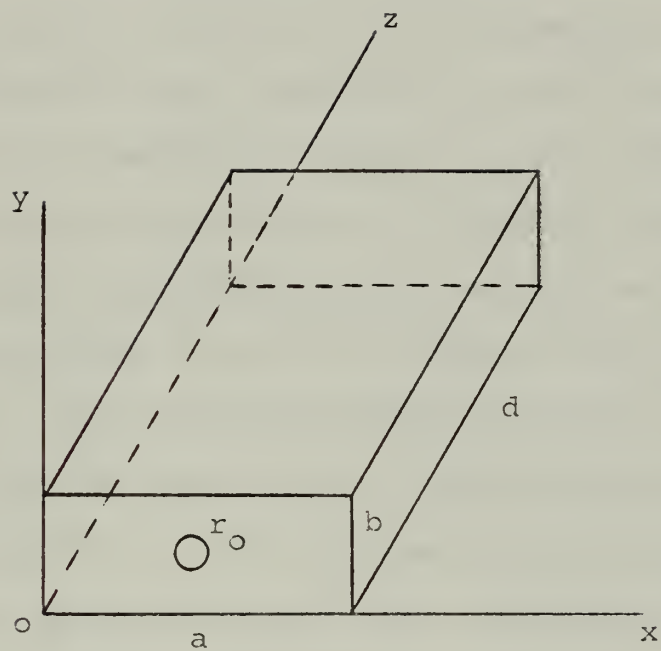


Figure (2-6). Coupled Waveguide Cavity

III. EQUIVALENT CIRCUIT FOR THE APERTURE COUPLED CAVITY

The effects of the coupling of a cavity resonator to an associated system are generally difficult to treat quantitatively. Any attempt to analyze the electromagnetic fields is usually discouraged by the complexity of most microwave systems. The periodic waveguide treated in this paper is a particularly interesting example of a coupled cavity system. In the first place, transmission-line methods may be used to construct an equivalent circuit to represent the aperture coupled cavity. The resonance properties of the original structure can then be described in terms of those of the cavity equivalent circuit. The equivalent circuit parameters can be determined by correlating the input impedances of the resonant cavity and a series resonant circuit [11].

A unit cell of the original periodic structure is shown in Fig. (2-6). It consists of a section of rectangular waveguide terminated in a short circuit. The width and height of the waveguide are indicated by a and b , respectively. The circular aperture, of radius r_0 , is centered in a conducting wall a distance d from the termination. If the losses associated with the aperture plane are small compared with those in the waveguide walls, the input impedance Z_c of the short-circuited section (not including the aperture plane), at $z = 0$, is given by [11]:

$$Z_c = Z_0 \tanh(\alpha + j\beta_g)d \quad (3-1)$$

Where Z_0 is the characteristic impedance of the waveguide, α is the attenuation constant, β_g is the waveguide propagation factor, and d is the length of the cavity.

Substituting a trigonometric identity for $\tanh(\alpha + j\beta_g)d$,

$$Z_c = Z_0 \frac{\tanh(\alpha d) + \tanh(j\beta_g d)}{1 + \tanh(\alpha d)\tanh(j\beta_g d)} \quad (3-2)$$

For a good conductor α is small, and for $\alpha d \ll 1$,

$$\tanh(\alpha d) \approx \alpha d, \quad (3-3)$$

thus

$$Z_c \approx Z_0 \frac{\alpha d + j \tan(\beta_g d)}{1 + j(\alpha d) \tan(\beta_g d)} \quad (3-4)$$

The length of the waveguide section is specified such that near resonance,

$$d \approx \frac{p\lambda_g}{2} \quad p = 1, 2, \dots \quad (3-5)$$

where λ_g is the guide wavelength.

Thus,

$$\beta_g d \approx p\pi, \quad p = 1, 2, \dots \quad (3-6)$$

Since $\tan(\beta_g d) = -\tan(p\pi - \beta_g d)$, and $|p\pi - \beta_g d| \ll 1$,

Eq. (3-4) becomes

$$Z_c \approx Z_0 \frac{\alpha d + j(\beta_g d - p\pi)}{1 + j\alpha d(\beta_g d - p\pi)} \quad (3-7)$$

The behavior of the cavity for frequencies near resonance is of primary interest, therefore expanding Eq. (3-7) in a Taylor series about $\omega = \omega_0$,



$$Z_c \approx Z_c \big|_{\omega=\omega_0} + \frac{\partial Z_c}{\partial \omega} \big|_{\omega=\omega_0} (\omega - \omega_0) \quad (3-8)$$

plus higher order terms which are assumed to be negligible.

Letting $Z_c = Z_0 z_c$, Eq. (3-8) can be expressed as:

$$z_c \approx z_c \big|_{\omega=\omega_0} + \frac{\partial z_c}{\partial \omega} \big|_{\omega=\omega_0} (\omega - \omega_0) \quad (3-9)$$

The waveguide propagation constant is a function of frequency and the particular propagating mode, and is given by:

$$\beta_{gnm}^2 = \left(\frac{\omega}{c}\right)^2 - \left[\left(\frac{n\pi}{a}\right)^2 + \left(\frac{m\pi}{b}\right)^2\right] \quad (3-10)$$

where n and m are mode numbers, a and b are waveguide dimensions, ω is the particular operating frequency, and c is the velocity of light.

To evaluate the terms in Eq. (3-9), the following chain rule is used:

$$\frac{\partial z_c}{\partial \omega} = \frac{\partial z_c}{\partial \beta_g} \cdot \frac{\partial \beta_g}{\partial \omega}$$

Equation (3-9) reduces to the following expression where the value of β_g near resonance, given by Eq. (3-6), is substituted:

$$z_c \approx Z_0 \alpha d + Z_0 j d (1 - \alpha^2 d^2) \frac{\partial \beta_g}{\partial \omega} \quad (3-11)$$

and from Eq. (3-10),

$$\frac{\partial \beta_g}{\partial \omega} = \frac{\omega d}{p \pi c^2} \quad (3-12)$$

Since $\alpha d \ll 1$, Eq. (3-11) reduces to

$$Z_c \approx Z_0 \alpha d + j \frac{Z_0 \omega_0 d^2}{p \pi c^2} (\omega - \omega_0) \quad (3-13)$$

Equation (3-13) is of the same form as the expression for the input impedance of a series RLC circuit which, in the frequency domain, is given by:

$$Z_S = R + j(\omega L - \frac{1}{\omega C}) \quad (3-14)$$

At resonance,

$$\omega_o^2 = \frac{1}{LC}$$

or

$$L = \frac{1}{\omega_o^2 C} \quad (3-15)$$

$$\text{therefore } Z_S = R + j \frac{(\omega^2 - \omega_o^2)}{\omega_o^2 \omega C} \quad (3-16)$$

For frequencies in the neighborhood of resonance, $\omega \approx \omega_o$ and $\omega + \omega_o \approx 2\omega_o$.

Therefore Eq. (3-16) can be approximated by:

$$Z_S \approx R + j 2L(\omega - \omega_o) \quad (3-17)$$

When the corresponding coefficients of $(\omega - \omega_o)$ in Eqs. (3-13) and (3-17) are equated, the following expressions for the equivalent circuit parameters can be identified [1]:

$$\begin{aligned} R &= \alpha d Z_o \\ L &= \frac{\omega_o d^2 Z_o}{2p\pi c^2} \\ C &= \frac{2\pi c^2}{p\omega_o^3 d^2 Z_o} \end{aligned} \quad (3-18)$$

The resonant angular frequency ω_o in Eq. (3-18) is defined by the characteristic equation of the waveguide given by Eq. (3-10).

It is important to realize this series equivalent circuit is a valid representation of the waveguide cavity for a particular TE_{nmp} or TM_{nmp} mode.

The resistance term given in Eq. (3-18) represents the losses in the waveguide cavity. Most waveguide structures have a very high Q , so the effect losses have on resonance calculations is negligible [5].

The impedance properties possessed by coupling apertures were discussed briefly in Section I. The aperture is represented in the equivalent circuit by an inductor in shunt with the other components. It is designated L_a and has a value of susceptance b_a that is a function of the cavity dimensions and operating frequency. [10].

The equivalent circuit for the resonant cavity and coupling aperture is shown in Fig. (3-1).

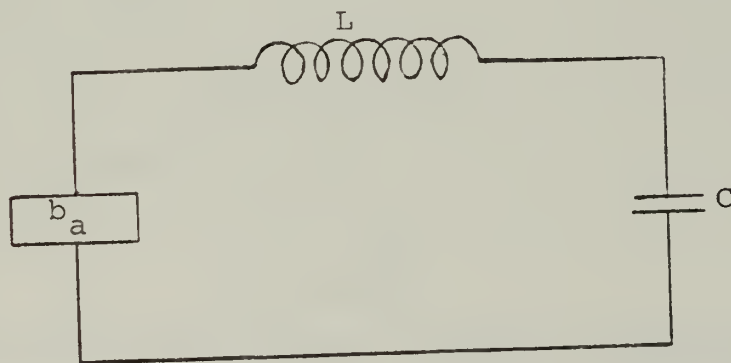


Figure (3-1). Equivalent circuit for an Aperture coupled waveguide cavity.

The resonant frequency of the aperture coupled cavity corresponds to the resonant frequency of the equivalent circuit shown in Fig. (3-1). This frequency is designated ω'_0 . Thus

$$\omega'^2_0 = \frac{1}{(L_a + L)C} \quad (3-19)$$

From Eq. (3-15)

$$C = \frac{1}{\omega^2_0 L}$$

Therefore

$$\omega'^2_0 = \frac{\omega^2_0}{L(1 + \frac{L_a}{L})} \quad (3-20)$$

For $L \gg L_a$, Eq. (3-20) can be approximated by

$$\omega'^2_0 \approx \omega^2_0 (1 - \frac{L_a}{L}) \quad (3-21)$$

Using the binomial expansion, Eq. (3-21) can be further reduced to the following expression:

$$\omega'_0 \approx \omega_0 (1 - \frac{L_a}{2L}) \quad (3-22)$$

$\Delta\omega$ is defined as the change in resonant frequency of the coupled cavity. Thus,

$$\Delta\omega \equiv \omega_0 - \omega'_0 \quad (3-23)$$

Substituting ω'_0 from Eq. (3-22),

$$\Delta\omega = \omega_0 \frac{L_a}{2L} \quad (3-24)$$

With the value of L given by Eq. (3-18), $\Delta\omega$ becomes:

$$\Delta\omega = - \frac{p\pi\omega_0}{b_a} \left(\frac{c^2}{\omega_0 d} \right)^2 \quad (3-25)$$

where b_a is the normalized susceptance representing the coupling aperture.

With known cavity specifications and aperture properties, the frequency shift of a coupled cavity can now be determined quantitatively by Eq. (3-25). The problem of determining the coupling coefficients of the original structure, which are defined in terms of this frequency shift, has thus been accomplished.

IV. THEORETICAL CALCULATIONS AND LABORATORY RESULTS

The method of analysis presented in the preceding sections, as well as the method presented in Appendix A, was developed for a rectangular waveguide cavity with a circular coupling aperture. In this section the results obtained by applying these methods to a practical model are presented. The resonance properties of this cavity model were measured in the laboratory and are also included.

The cavity model considered is shown in Fig. (2-6), where the indicated dimensions are as follows:

$$a = 2.29 \text{ cm}; b = 1.02 \text{ cm}; d = 7.62 \text{ cm}; r = 0.48 \text{ cm}.$$

The cavity was assumed to be resonating in the TE_{103} mode. For the specified dimensions, the unperturbed resonant frequency, f_{103} , was determined using Eq. (2-1). The calculated value is given below.

$$f_{103} = 8828 \text{ MHz}$$

A. EQUIVALENT CIRCUIT METHOD

Referring to Eq. (3-25), the only unknown quantity is the normalized susceptance of the circular aperture. This quantity is a function of the frequency and the dimensions of the cavity and aperture. For this cavity model, the following value was determined from existing data [10]:

$$b_a = -5.5$$

Therefore the change in resonant frequency Δf , as determined by Eq. (3-25), is

$$\Delta f = 76 \text{ MHz}$$

B. EQUIVALENT DIPOLE METHOD

The frequency shift for the same cavity model was next determined by using Eq. (A-8) given in Appendix A. All quantities in this expression are known, thus a straightforward calculation results in the following value of Δf ,

$$\Delta f = 66 \text{ MHz}$$

C. EXPERIMENTAL MEASUREMENTS

A cavity with the above dimensions was constructed in the laboratory. The cavity was coupled to a power source through a circular aperture with a radius as specified. The resonant frequency of this coupled cavity was measured using the equipment shown in block diagram form in Fig. (4-1).

The frequency of the generated signal was swept through the unperturbed resonant frequency, f_{103} , given above. By observing the cavity's resonance characteristics on the oscilloscope, the frequency corresponding to the perturbed TE_{103} mode was determined. This frequency, designated f'_{103} , was measured using the wavemeter. The value measured is given below:

$$f'_{103} = 8766 \text{ MHz}$$

The frequency shift, Δf , was determined by the following relation:

$$\Delta f = f_{103} - f'_{103}$$

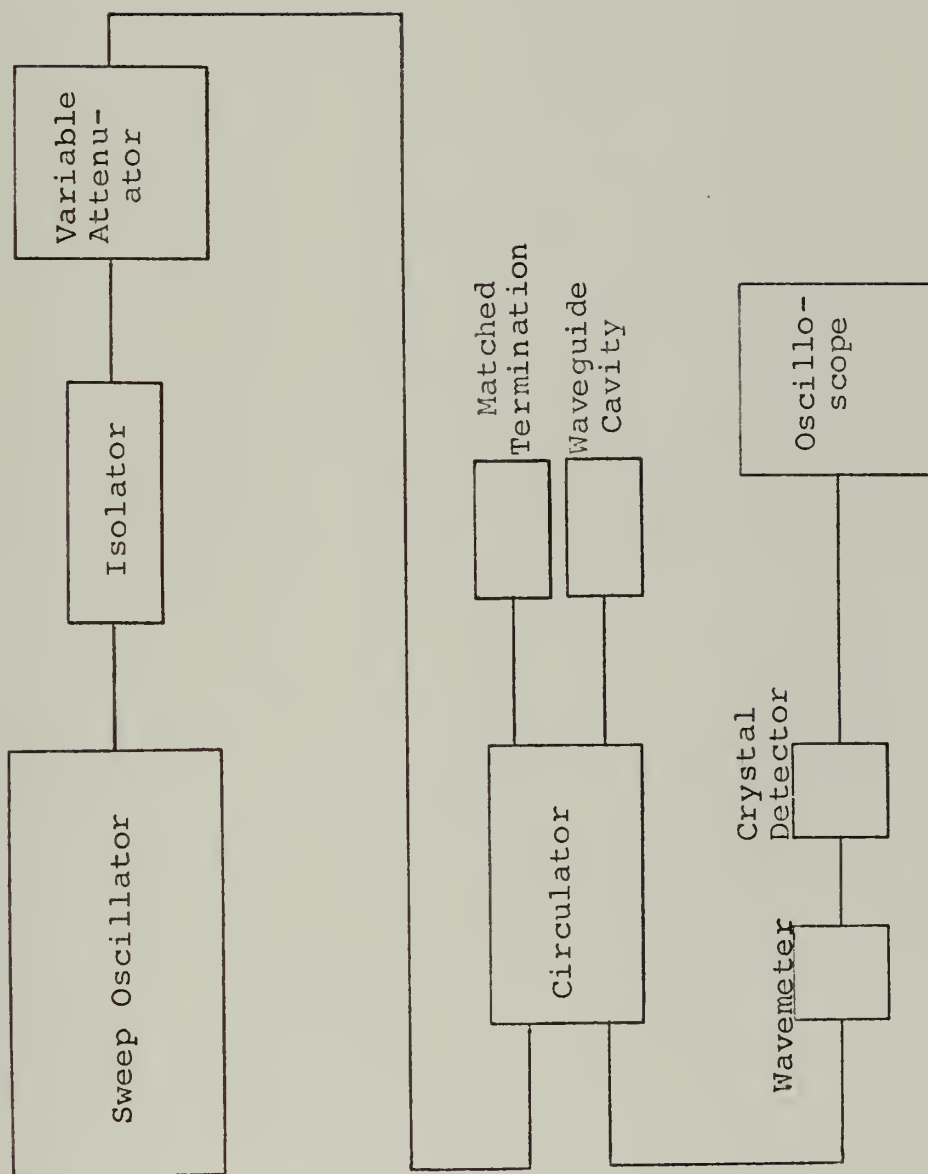


Figure (4-1). Equipment diagram for measuring the resonant frequency of a coupled cavity.

Therefore

$$\Delta f = (8828 - 8766) \text{ MHz}$$

or

$$\Delta f = 62 \text{ MHz}$$

V. CONCLUDING REMARKS

A basis has been presented for the evaluation of the coupling coefficients associated with periodic structures of the coupled cavity type. The problem was approached by representing the unit cell suggested by the coupled mode theory, by an equivalent circuit. The measure of coupling was then determined by the resonant frequency of this circuit.

This method of analysis is attractive in that it may be applied to any coupled cavity structure where the waveguide is uniform and the susceptance of the coupling aperture is known. Assuming a dominant mode of propagation, the construction of the equivalent circuit is simplified by using transmission line techniques.

The results of this method, as given in Section IV, indicate a certain discrepancy when compared to those of the other two methods presented. This does not render the equivalent circuit method useless, but does indicate that some judgment be used in certain microwave applications.

Certainly the beauty of this method lies in its simplicity and physical appeal. The rigorous field analysis problem is reduced to one involving basic circuit concepts.

Haus [6] has derived the equations describing the coupled modes of propagation by using a variational approach. It is recommended, as a topic for further study, that this method be applied to the coupled modes of oscillation.

APPENDIX A

REPRESENTING THE CIRCULAR APERTURE BY DIPOLES

A method is presented for evaluating the coupling parameters associated with a coupled-cavity structure in terms of the "polarizability" of the circular aperture.

The fundamental ideas concerning small apertures and their coupling properties were developed by Bethe [2]. His work has been the source for much of the existing material concerning aperture coupling in cavity and waveguide applications.

Bethe has shown that the fields in the vicinity of a small circular aperture in a conducting wall can be closely approximated by the sum of the unperturbed fields and those produced by electric and magnetic dipoles. The equivalent electric dipole is oriented normal to the aperture and has a strength proportional to the normal electric field. Similarly, the equivalent magnetic dipole is in the plane of the aperture with a strength proportional to the tangential magnetic field. The constants of proportionality, α_e and α_m , are called the electric and magnetic polarizabilities, respectively. They characterize the coupling properties of the aperture and are dependent on the aperture geometry. A qualitative argument to demonstrate the physical reasonableness of these properties of an aperture is given below.

Consider the aperture plane of a single cavity that supports a TE_{10p} mode, where p is an integer. The normal electric field at the aperture plane is zero, therefore there is no induced electric dipole. There does exist a tangential magnetic field, for which there is an equivalent magnetic dipole. To illustrate this idea, consider Fig. (A-1). In Fig. (A-1a) the tangential magnetic field is indicated in the absence of an aperture. With the aperture present, the impinging field lines will fringe through, as shown in Fig. (A-1b). These fringing field lines approximate those produced by the magnetic dipole shown in Fig. (A-1c). The presence of the aperture also perturbs the field on the incident side of the wall. Collin has shown, by an argument based on image theory, that the effect of the conducting wall is equivalent to removing the wall and doubling the strength of the dipole [4].

These equivalent dipoles correctly account for the fields coupled through the aperture in the conducting wall. The energy associated with these fields is directly related to the resonant frequency of the cavity by Eq. (2-2), which is repeated here.

$$\frac{\Delta\omega}{\omega_0} = \frac{\Delta W}{W_0} \quad (A-1)$$

where ΔW represents the change in stored energy introduced by the aperture and W_0 is the total stored energy of the unperturbed cavity.

The right side of Eq. (A-1) can be expressed by the following integral relationship [7]:

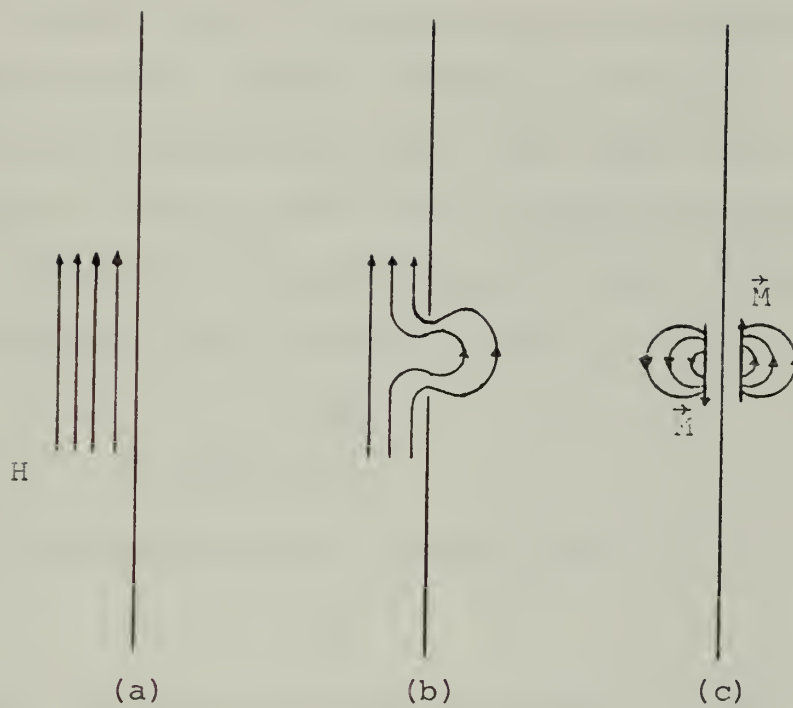


Figure (A-1). Aperture in a conducting wall.

$$\frac{\Delta W}{W_0} = - \frac{1/4 \iiint (\vec{P}^* \cdot \vec{E} + \vec{M}^* \cdot \vec{B}) dv}{1/4 \iiint (\epsilon \vec{E}^* \cdot \vec{E} + \mu \vec{H}^* \cdot \vec{H}) dv} \quad (A-2)$$

where ϵ and μ are the permittivity and permeability of the medium, \vec{P} and \vec{M} are the electric and magnetic dipole moments per unit volume, \vec{E} and \vec{H} represent the unperturbed cavity fields, and the (*) denotes a complex conjugate. The upper integral can be simplified by the fact that for a TE_{10p} mode, there is no E - field normal to the aperture plane, thus \vec{P} is zero. Also, for the small-coupling case, \vec{M} can be defined by the following limit expression [7]:

$$\vec{M} \equiv \lim_{\Delta v \rightarrow 0} \frac{\sum_i \vec{m}_i}{\Delta v} \quad (A-3)$$

where \vec{m} , the dipole moment, is given by

$$\vec{m} = - \alpha_m \vec{H} \quad (A-4)$$

For a small circular aperture with radius r_0 , the magnetic polarizability α_m is given by [7]:

$$\alpha_m = \frac{4}{3} r_0^3 \quad (A-5)$$

Thus, ΔW , in Eq. (A-2), can be reduced to the following

$$\Delta W = - \frac{2}{3} r_0^3 \mu |\vec{H}|^2 \quad (A-6)$$

W_0 , the unperturbed cavity energy, can be determined by a straightforward field analysis. See [4, 5]. For the rectangular waveguide cavity shown in Fig. (2-2), resonating in the TE_{10p} mode, W_0 can be expressed as:

$$W_{10p} = \frac{1}{2} \mu |\vec{H}|^2 abd \left(\frac{p^2 a^2 + d^2}{d^2} \right) \quad (A-7)$$

where a, b, and d are the cavity dimensions indicated in Fig. (2-2).

By substituting Eqs. (A-6) and (A-7) into Eq. (A-1), the following expression for the shift in frequency of the coupled cavity resonating in the TE_{10p} mode is obtained:

$$\Delta\omega = \frac{\frac{4}{3} r_o^3 \omega_o}{(abd) \left(\frac{p^2 a^2 + d^2}{d^2} \right)} \quad (A-8)$$

Equation (A-8), like Eq. (3-25), allows one to compute the coupling coefficients associated with a coupled cavity structure in terms of the resonant frequency of a unit cell of that structure.

LIST OF REFERENCES

1. Atwater, H. A., Introduction to Microwave Theory, p. 146-151, McGraw-Hill, 1962.
2. Bethe, H. A., "Theory of Diffraction by Small Holes," Physics Review, v. 66, p. 163-182, 1 October 1944.
3. Belohoubek, E., "Propagation Characteristics of Slow-Wave Structures Derived from Coupled Resonators," RCA Review, v. 19, p. 283-290, June 1958.
4. Collins, R. E., Foundations for Microwave Engineering, p. 321-340, McGraw-Hill, 1966.
5. Ghose, R. N., Microwave Circuit Theory and Analysis, p. 182-190, McGraw-Hill, 1963.
6. Massachusetts Institute of Technology Research Laboratory of Electronic Report 316, Electron Beam Waves in Microwave Tubes, by H. A. Haus, 8 April 1958.
7. Johnson, C. C., Field and Wave Electrodynamics, McGraw-Hill, 1965.
8. Knorr, J. B., "Coupling of Modes of Oscillation," Unpublished.
9. Louisell, W. H., Coupled Mode and Parametric Electronics, p. 1-32, Wiley, 1960.
10. Marcuvitz, N., Waveguide Handbook, McGraw-Hill, 1951.
11. Montgomery, C. G., Dicke, R. H., and Purcell, E. M., Principles of Microwave Circuits, p. 231-235, McGraw-Hill, 1948.

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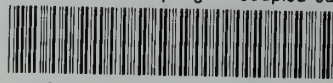
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13. ABSTRACT <p>A method is presented for evaluating the coupling coefficients associated with loosely coupled cavity structures. The measure of coupling is expressed in terms of the shift in resonant frequency exhibited by a single aperture-coupled cavity. The frequency shift is determined from the resonant frequency of an equivalent circuit representing the aperture-coupled cavity. This method of analysis is applied to a specific waveguide cavity, coupled by a small circular aperture.</p> <p>The results of this method are then presented for comparison with those obtained by two other techniques.</p>			

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